**CIS 522: Homework 1**

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Q1.) Chapter 1, Exercise 4, Page 23

Ans.) **Brief Description**: The basic idea of the Gale-Shapley(1962) Algorithm is to, in this case, find a way to assign each student to *at most* one hospital in such a way that all the vacant positions in the list of ‘m’ hospitals is filled by a student chosen from a subset of ‘n’ students. Concurrently, both the hospitals and the students have an ordered list of preferences.

Now, in this case, there is a slight caveat given that each student is looking to accept a position in exactly one hospital whereas each hospital is looking to fill multiple positions. Also, the question in this case assumes that the number of graduating students outnumber the total available positions across ‘m’ hospitals. Hence, at any point in time a particular student is either “committed” or “free”. Similarly, a hospital either has available positions or it is “full”.

**Pseudocode**:

While some hospital ‘mi’ has available positions

mi offers a position to the next student ‘ni’ on it’s preference list

if ni is free then

ni accepts the offer

else(nj is already committed to a hospital mk )

if nj prefers mk to mi  then

nj  remains committed to mk

else nj  becomes committed to mi

the number of available positions at mk increases by 1

the number of available positions at mi decreases by 1

The algorithm, in this case, will have a time complexity of O(m\*n) unlike regular Gale-Shapley which has a time complexity of O(n2) owing to the possibility of a one-to-one matching in a square matrix.

**Proof: Algorithm returns a stable assignment** -> Extending from Lemma (1.5) wherein we prove that a set returned at termination is a perfect match, we will now use Lemma (1.6) to prove that the returned set is a stable match.

As the question stem lays out the necessary instability conditions, let us tackle them one-by-one:

1. Let there be two students s and s`, and a hospital h.

If h preferred s` to s, then h would have offered a position to s` before offering one to s. Consequently, s` would have a position at another hospital h` and would not be free at the termination of the algorithm. Hence, by contradiction, the first instability is disproved.

1. Let there be a pair (h`, s`) which is an unstable pairing.

Then it stands to argue that h` must have offered a position to s` in spite having ‘x’ residents whom h` prefers over s`.

Correspondingly, at some point, s` must have rejected h` for ‘h’ which he/she preferred and must therefore be committed to another hospital h`` (likely different from ‘h’) which he/she prefers over h`. Hence, by contradiction, we see that this pairing is actually stable.

Q2.) Implementation of Gale-Shapley algorithm

Ans.)

1. Victor to Zeus:

Text

Description automatically generated

1. Zeus to Victor:

Text

Description automatically generated

1. Amy to Erika:

Text

Description automatically generated